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Gravitational topological quantum field theory versus $N = 2$ $D = 8$ supergravity and its lift to $N = 1$ $D = 11$ supergravity

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Abstract: In a previous work, it was shown that the 8-dimensional topological quantum field theory for a metric and a Kalb-Ramond 2-form gauge field determines $N = 1$ $D = 8$ supergravity, at least in the fermion quadratic aproximation. It is shown here that, the combination of this TQFT with that of a 3-form determines $N = 2$ $D = 8$ supergravity, that is, an untruncated dimensional reduction of $N = 1$ $D = 11$ supergravity. Our construction holds for 8-dimensional manifolds with Spin(7) $\subset$ SO(8) holonomy. The relationship between the supergravity and topological actions is shown up to quartic fermionic terms, around a Spin(7) vacuum. This suggests that the origin of local Poincaré supersymmetry might be the geometrically well-defined gravitational topological symmetry. We also indicate a mechanism for the lift of the TQFT in higher dimensions, which generates Chern–Simons couplings.

Keywords: Topological Field Theories, Space-Time Symmetries, BRST Symmetry, Supergravity Models.

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1. Introduction

Poincaré supersymmetry and topological supersymmetry are deeply related on manifolds with special holonomies. This has been observed in [1] and [2], where the 8-dimensional Yang–Mills topological quantum field theory (TQFT) is constructed. [1] also suggests that the introduction of a TQFT for a 3-form in eight dimensions is of relevance for a better understanding of supergravities and for determining effective actions for the M theory.

[3] indicates the existence of a formal link between topological sigma-models and all types of anomaly-free superstrings. This might be a signal that the origin of local Poincaré supersymmetry is the gravitational topological symmetry. Then, an interesting question is whether $D = 11$ supergravity, which determines all known supergravities in lower dimensions as limits of superstrings, can be constructed in the context of a TQFT. Eventually, there is the possibility that there is a unique topological symmetry that can be represented in two possible phases, one which is purely topological and the other one which describes particles.

To concretely understand how a supergravity can be related to a gravitational TQFT is not a trivial task. We have already demonstrated that topological gravities in four and eight dimensions can be respectively untwisted into $N = 2$ $D = 4$ supergravity [4] and a truncated version of $N = 1$ $D = 8$ supergravity [5]. The TQFT of a general tensor of rank two in eight dimensions, identified as the symmetric metric and the Kalb–Ramond 2-form, determines in a twisted way the complete $N = 1$ $D = 8$ supergravity (at least in the quadratic fermion approximation) [6]. Moreover, introducing a TQFT for the 2-form in[6] clarifies questions opened in earlier work, which are related to the interpretation of graviphotons. In all these cases, one finds that a gravitational TQFT can be identified with an Euclidean supergravity, around the solution of a gravitational instanton. Let us notice that the case of $N = 1$ $D = 2$ supergravity is simplest and it can be shown from many different points of views that topological 2d-gravity can be untwisted into 2d-supergravity. We analyzed this in [7] and a geometrical insight on its topological 2-dimensional gravitational invariance was given in [8].

Thus, several examples show that the gravitino can be untwisted in topological ghosts for the reparametrization symmetry. It is an appealing feature, since it indicates that the gravitino can be identified as a curvature (in an enlarged space) rather than as a connection. Local supersymmetry is then a ghost of ghost symmetry rather than a gauge symmetry. This changing point of view eliminates many of the puzzling questions that are related to the elusive construction of a gauge group for the Poincaré local supersymmetry. Moreover, within the context of topological gravity coupled to TQFTs of forms, all elements of supergravity multiplets acquire robust geometrical definitions.

In this paper, we reach an earlier goal. We show that the field spectrum of $N = 1$ $D = 11$ supergravity can be determined in the context of a 8-dimensional gravitational TQFT. The equivalence of the supergravity and topological actions is shown up to quartic fermionic terms, around a Spin(7) invariant vacuum.

In [6], we had shown how to build $N = 1$ $D = 8$ supergravity as a TQFT for a metric and a 2-form. Here, by using a freedom in the ghost sector of the Kalb–Ramond 2-form that we had not exploited in [6], we enlarge the ghost of ghost symmetry of the twisted $N = 1$ $D = 8$ supergravity. This suggests us to introduce new degrees of freedom, by defining a TQFT for a 3-form that we couple to the TQFT of [6]. As a consequence, a second gravitino emerges, which is made of some of the topological ghosts of the 3-form. Remarkably, the resulting theory turns out to rely on the field spectrum of $N = 2$ $D = 8$ supergravity, whose (untwisted) classical field content is the metric, a 3-form, three 2-forms, seven scalars, two gravitinos and six Majorana spinors. Thus, we will find the field spectrum of our TQFT is an untruncated dimensional reduction of the spectrum $N = 1$ $D = 11$ supergravity, in a twisted form. This technical result has of course some interest, although its justification is yet not obvious.

In order to define the TQFT and perform in the easiest way the twist operations, we consider 8-dimensional manifolds with $\mathrm{Spin}(7) \subset \mathrm{SO}(8)$ holonomy. (We could as well choose a smaller holonomy group, for instance G_2.) The covariantly constant spinor η that exists in such manifolds is actually sufficient to relate forms and spinors by "twist". In particular, it determines a closed self-dual 4-forms $\eta\gamma_{[\mu}\gamma_\nu\gamma_\rho\gamma_{\sigma]}\eta$ that is essential for defining as in the 8d Yang–Mills TQFT self-duality equations, which are generally invariant only under the action of the holonomy group.

So, to summarize, what we will show is a relationship between a gravitational TQFT and the $N = 2$ $d = 8$ supergravity theory, which one expands around a gravitational instanton. The "twist" operation is what changes the fermions of the TQFT, which exactly balance all contributions of bosons, modulo zero modes, into fermions that can be interpreted as particles, since they satisfy the physical spin-statistic relation.[1]

[1]It should me understood that the geometrical interpretation of the twist generally depends on the

After untwisting the TQFT, it is worth noting that, as often, one recovers the full Lorentz invariance (in 8 dimensions). Indeed, the untwisting is such that the explicit dependence of the TQFT on the constant spinor is absorbed in the change of variables that maps forms on spinors, when one switches to the variables that are relevant for supergravity. Then, we get the supergravity action in its SO(8) invariant form (in the fermionic quadratic approximation). This result generalizes that found for the topological Yang–Mills theory in 8 dimensions.

An intriguing question is whether the covariantly constant spinor, which makes the twist possible, has a physical origin, for instance, as an expectation value of some field. In the presence of a brane, a form may exist in higher dimensions with a constant flux φ through a hypersurface, which can generate the constant spinor ϵ, by relations like ${}^t\epsilon\gamma^a \ldots \gamma^c\ \epsilon = \varphi^{a \cdots c}$. We suggest another possibility, where one may view this spinor as an expectation value of the commuting Faddeev–Popov ghost of local supersymmetry.

The paper is organized as follows.

Section 2 displays heuristic arguments for tentatively explaining the possibility of obtaining the spectrum of supergravity within the context of gravitational gravity. Section 3 details our precise arguments, which show the relationship between the field spectrum of the 8-dimensional gravitational TQFT for a metric, a 2-form and a 3-form and that of $N = 2$ $D = 8$ supergravity. The TQFT spectrum is such that the supergravity spectrum is obtained after field eliminations, stemming from algebraic equations of motion in the TQFT. The TQFT action is constructed directly as a BRST-exact term. Section 3 also shows that the actions of both theories are identical in the quadratic fermionic aproximation, up to twist. Section 4 sketches an argument that new TQFTs seem to exist in eight and seven dimensions, which involve as dynamical fields a twisted gravitino and forms, but not a metric. This reflects the well-known existence of a supergravity multiplet with a single gravitino but not a metric. Section 5 shows a mechanism for lifting the theory in higher dimensions and generating Chern-Simons terms.

2. Heuristic arguments for explaining the TQFT-supergravity correspondence

Superstring theory induces gravity, and is explicitly reparametrization invariant in target-space. The way supergravity and target-space supersymmetry emerge is less direct. Superconformal invariance on the world-sheet is indeed the fundamental symmetry that selects vertex operators for obtaining the gravitino in the quantum field theory limit, and target-space supersymmetry is only discovered at the end of the construction.

Our knwowledge of target space local supersymmetry basically amounts to infinitesimal transformations. They build an open “algebra ” that closes only modulo equations of motions, which makes its geometrical interpretation obscure. Large supersymmetry transformations, if any, are unknown.

dimension.

It is thus very difficult to interpret the gravitino as a gauge field and the algebra of supergravity transformations as generating a gauge symmetry group. The TQFT formulation actually provides a substitute, since, as we will see, it identifies the gravitino as a (twisted) topological ghost, which is a curvature rather than a gauge field, the graviphotons as ghosts of ghosts, etc .... The reason why topological ghosts have the interpretation of a curvature is basically that they derive from the action of exterior operators upon fields that have the status of connection, as explained, e.g., in the Yang-Mills case [1]. The gravitational TQFT construction may thus appear as a good substitute that allows one to skip the construction of a group out of infinitesimal supersymmetry transformations of supergravity. In what follows, we will try to heuristically justify the existence of such a possibility of replacing the principle of local supersymmetry by that of gravitational topological symmetry.

We start by consider superstring theory in the NSR formalism. The string coordinate X^μ has world-sheet supersymmetric partners ψ^μ and $\bar\psi^\mu$, where μ labels the world index of target-space. The superstring field $(\psi^\mu, \bar\psi^\mu)$ is a world-sheet spinor. Once a conformal structure has been chosen for the world-sheet of the string, both generators Q and $\bar Q$ of world-sheet supersymmetry act as follows on the string field:

$$QX^\mu = \psi^\mu + \cdots, \qquad \bar Q X^\mu = \bar\psi^\mu + \cdots \tag{2.1}$$

$(\psi^\mu, \bar\psi^\mu)$ has the physical spin-statistic relation on the world-sheet, but the unphysical one in target-space. Consider now the the induced metric $G_{\mu\nu}$ of the target-space quantum field theory limit. Eq. (2.1) suggests that there is a topological symmetry for X^μ, by interpreting ψ^μ as a topological ghost associated to arbitrary local variations of X on the worldsheet, over which one eventually performs path integral. On the other hand, $G_{\mu\nu}$ can be understood as a functional of X, and, when the details of the worldsheet have been washed out, eq. (2.1) also suggests that $G_{\mu\nu}$ might have a symmetry of the type:

$$QG_{\mu\nu} = \psi_{\mu\nu} + \cdots, \tag{2.2}$$

where $\psi_{\mu\nu}$ is a topological ghost that correspond to arbitrary variations of $G_{\mu\nu}$, which are themselves induced by those of X. If we now forget the string theory origin of he field $G_{\mu\nu}$, $\psi_{\mu\nu}$ clearly looks as a target-space topological ghost, and we may consider eq. (2.2) as defining a fundamental symmetry in quantum field theory, namely gravitational topological invariance. The spin-statistic relation for $\psi_{\mu\nu}$ is unphysical, so that it contributes negatively to the energy, with contributions that are opposite to those of the metric. If we succeed (as we will do) in building a TQFT based on eq. (2.2), it should be related one way or the the other to supergravity, since the later is the well know limit of superstring theory, which was our starting point. This observation justifies our quest of trying to formulate $N = 1$ $D = 11$ supergravity as a TQFT, when it is expressed under the form of its dimensional reduction in 8 dimensions.

And, indeed, after having achieved the details of the construction of the TQFT for a metric, a 2-form and a 3-form in 8 dimensions, we will have (in the quadratic fermion aproximation) the desired identification between a gravitational topological BRST algebra and the infinitesimal symmetries of $N = 2$ $D = 8$ supergravity,, which is an expression of the

$N = 1$ $D = 11$ supergravity. It is striking that, in this way, all ingredients of supergravity will be described from geometrical considerations in the space of field configurations of a metric, a 2-form and a 3-form in 8 dimensions.

The existence of the twist between supergravity and topological gravity may thus be a natural feature, due to the existence of a twist between the topological sigma model (coupled to 2d topological gravity) and superstrings (coupled to $N = 1$ 2d supergravity). The way one interprets the operators Q and $\bar{Q}$ as BRST and anti-BRST operators for the topological sigma-model is by shifting to zero value the conformal weight of the superstring fields ψ^μ and $\bar{\psi}^\mu$. To compensate for the change in conformal charge, one also performs a compensating twist for the worldsheet local supersymmetry ghosts that transforms the world-sheet $N = 1$ supergravity into 2-dimensional topological gravity. Such a twist can be seen as a mere change of variables, as for instance indicated in [3, 7]. Since this twisted form of superstring theory looks completely topological, it is plausible that one of its quantum field theory limits be topological gravity instead of supergravity, with some link between both theories that is a reminder of the twist operation at the level of the worldsheet theory.[2]

These intriguing observations have been the support of our idea that supersymmetry in target-space is more of a topological origin than is usually expected. The next section is to show in detail how the spectrum of maximal supergravity, and its action, is actually related to a topological model of the Donaldson-Witten type [9].

3. Determination of the spectrum of $N = 2$ $D = 8$ supergravity

3.1 Reminder of the TQFT for $N = 1$ $D = 8$ supergravity

In [6], we have constructed the TQFT for a tensor of rank 2 for manifolds in eight dimensions with Spin(7) holonomy. We obtained that the BRST topological multiplet is

[2]The proposal that a precise description of target-space symmetry does not need need to be precised at an early stage of the construction of the theory is reinforced by observations presented in [3]. There, we have shown that, for a genuine world-sheet theory, i.e, a pure 2d-surface theory without matter and not embedded in a target space, but with a rich enough 2-dimensional gravitational structure for enforcing the absence of a conformal anomaly, one can extract target-space coordinates from the world-sheet structure. Indeed, if we take 2d-supergravity with supersymmetry of rank larger than four, the sum of the contributions of all its ghosts to the conformal anomaly vanishes and it is thus inconsistent to introduce matter under the form of external string coordinates. The latter would generate an anomaly. On the other hand, we have shown that we can perform various twists of the 2d-ghosts, while keeping the conformal anomaly equal to zero and recover all possible superstring theories, with $N < 4$ worldsheet supersymmetry, combined with decoupled TQFTs [3]. In this presentation, the physical string coordinates, i.e, the coordinates of the effective target-space, appear as bound states of the additional ghosts that are initially introduced for describing the extended supersymmetry on the world-sheet. The superstring coordinates are defined in an analogous way. An alternative scheme is to replace the 2d-supergravity of rank N by the topological W_N gravity, with the interesting limiting case of W_∞ gravity, which has a Lie algebra structure. The vertex operators that describe the fields of the limiting supergravity theory are then composites of 2-dimensional ghost fields that arise from a pure two-dimensional geometrical structure, with no early reference to a target-space. Eventually, the defining equation of the symmetry in target space should be as in eq. (2.1).

basically:

$$e_\mu^a, B_{\mu\nu}, \sigma, A_\mu^{(2)}, A_\mu^{(-2)}, \left(\Psi_\mu^{(1)a}, \bar\Psi_\mu^{(-1)ab^-}, \bar\Psi_\mu^{(-1)}\right), \left(\bar\chi_\mu^{(-1)}, \Psi_{\mu\nu^-}^{(1)}, \chi^{(1)}\right). \tag{3.1}$$

This is nothing but the spectrum of $N = 1$ $D = 8$ supergravity, up to a twist, which is enabled by the existence of a covariantly constant spinor in the manifold.

In more detail, we constructed in [6] a TQFT for the vielbein e_μ^a, the spin connection ω_μ^{ab} and the Kalb-Ramond 2-form $B_{\mu\nu}$. We found that the topological BRST multiplets for these fields can be expressed as follows:

$$\begin{array}{ccccccccc}
 & & & & e_\mu^a & & & & \\
 & & & \swarrow & & & & & \\
 & & \Psi_\mu^{(1)a} & & & & (\bar\Psi_\mu^{(-1)ab^-}, \bar\chi_\mu^{(-1)}) & & \\
 & \swarrow & & & & \swarrow & & & \\
\Phi^{(2)a} & & & & \sigma, \Phi^{(0)ab^-}, (b_\mu^{(0)ab^-}, b_\mu^{(0)}) & & & & \bar\Phi^{(-2)a} \\
 & & & \swarrow & & & & \swarrow & \\
 & & \chi^{(1)}, \eta^{(1)ab^-} & & & & \bar\eta^{(-1)a} & &
\end{array} \tag{3.2}$$

$$\begin{array}{ccccccccc}
 & & & & \omega_\mu^{ab} & & & & \\
 & & & \swarrow & & & & & \\
 & & \tilde\Psi_\mu^{(1)ab} & & & & \bar{\tilde\Psi}_\mu^{(-1)ab} & & \\
 & \swarrow & & & & \swarrow & & & \\
\tilde\Phi^{(2)ab} & & & & \tilde\Phi^{(0)ab^-}, \tilde b_\mu^{(0)ab^-} & & & & \bar{\tilde\Phi}^{(-2)ab^+} \\
 & & & & & & & \swarrow & \\
 & & & & & & \bar{\tilde\eta}^{(-1)ab^+} & &
\end{array} \tag{3.3}$$

$$\begin{array}{ccccccccccc}
 & & & & & B_{\mu\nu} & & & & & \\
 & & & & \swarrow & & & & & & \\
 & & & \Psi_{\mu\nu}^{(1)} & & & \bar\Psi_{\mu\nu^-}^{(-1)} & & & & \\
 & & \swarrow & & & & \swarrow & & & & \\
 & A_\mu^{(2)} & & & (A^{(0)}, A_{\mu\nu^-}^{(0)}), b_{\mu\nu^-}^{(0)} & & & & A_\mu^{(-2)} & & \\
 \swarrow & & & & \swarrow & & & \swarrow & & & \\
R^{(3)} & & S^{(1)}, & (\Psi_{\mu\nu^-}^{(1)}, \Psi^{(1)}) & & & \bar S^{(-1)}, \bar\Psi_\mu^{(-1)} & & & & \bar R^{(-3)} \\
 & & \swarrow & & & \swarrow & & & & \swarrow & \\
 & & b_{S^{(1)}}^{(2)} & & & b_{\bar S^{(-1)}}^{(0)} & & & & b_{\bar R^{(-3)}}^{(-2)} &
\end{array} \tag{3.4}$$

To gain control on the gauge invariance of the graviphoton $A_\mu^{(-2)}$, we need a BRST quartet that includes an abelian ghost $d^{(-1)}$. We thus extend the pair $(A_\mu^{(-2)}, \Psi_\mu^{(-1)})$ in eq. (3.4) as follows:[3]

$$
\begin{array}{ccccccccc}
 & A_\mu^{(-2)} & \quad\hookrightarrow\quad & & & & A_\mu^{(-2)} & & \\
\swarrow & & & & & \swarrow & & & \\
\bar\Psi_\mu^{(-1)} & & & & \bar\Psi_\mu^{(-1)}, d^{(-1)} & & & & \\
 & & & \swarrow & & & & & \\
 & & \Phi^{(0)} & & & & & \bar\Phi^{(0)} & \\
 & & & & & & \swarrow & & \\
 & & & & & \bar\eta^{(1)} & & &
\end{array}
\tag{3.5}
$$

We also have the usual ghost systems for the reparametrization and Lorentz invariances:

$$
\begin{array}{ccccccc}
\xi^\mu & & \bar\xi^\mu & \qquad & \Omega^{ab} & & \bar\Omega^{ab} \\
 & \swarrow & & & & \swarrow & \\
 & b^\mu & & & & b^{ab} &
\end{array}
\tag{3.6}
$$

In these equations, some 2-forms have been decomposed in a Spin(7) invariant way as $X_{\mu\nu} = X_{\mu\nu^-} + X_{\mu\nu^+}$, where $X_{\mu\nu^-}$ and $X_{\mu\nu^+}$ are self-dual and antiself-dual projections of $X_{\mu\nu}$, with dimensions 7 and 21 respectively, according to $28 = 7 \oplus 21$. We have slightly improved our general notation of [6] by adding southwest arrows, which indicate which fields are related by topological BRST transformations.

For each field, the upper index in parenthesis indicates the ghost number. The fields that are not on the left edge of each pyramid come into topological pairs that are made of a commuting or an anticommuting antighost $\bar g$ with its Lagrange multiplier λ of the opposite statistics. They satisfy BRST equations that can be basically written as $s\bar g^{(g)} = \lambda^{(g+1)}$, $s\lambda^{(g+1)} = 0$, i.e, that are of a trivial type. These BRST equations can be improved in a way that expresses the reparametrization symmetry and the symmetries of forms in an equivariant way.

The fields that carry the essential geometrical information are on the left edge of each pyramid. We refer to [6], where the explicit BRST transformations of all above fields have been displayed in great details.

All topological BRST equations actually derive from geometrical equations on extended curvatures, which are of the following type (in the case of an abelian 2-form):

$$
(s+d)\left(B_{\mu\nu}dx^\mu \wedge dx^\nu + V_\mu^{(1)}dx^\mu + m^{(2)}\right) = \exp i_\xi \left(dB_2 + \Psi_{\mu\nu}^{(1)}dx^\mu \wedge dx^\nu + A_\mu^{(2)}dx^\mu + R^{(3)}\right).
\tag{3.7}
$$

The operator $\exp i_\xi$ generically takes into account the reparametrization symmetry. In this example of an abelian 2-form B_2, V_1^1 and m^2 are the standard ghost for the gauge symmetry $B_2 \sim B_2 + dV_1^1$ and $V_1^1 \sim V_1^1 + m^2$. The topological ghosts and ghosts of ghosts appear in the right hand side eq. (3.7). The expression of BRST equations as in eq. (3.7) is familiar

[3]The BRST symmetry for this U(1) invariance is expressed by:

$$
sA_\mu^{(-2)} = \bar\Psi_\mu^{(-1)} + \partial_\mu d^{(-1)}, s\bar\Psi_\mu^{(-1)} = -\partial_\mu \Phi^{(0)}, sd^{(-1)} = \Phi^{(0)}, s\Phi^{(0)} = 0, s\Phi^{(0)} = \bar\eta^{(1)}, s\bar\eta^{(1)} = 0\,.
$$

in the theory of descent equations for anomalies [11]. It encodes in a compact way all gauge symmetries of forms, including the ordinary gauge symmetries and reparametrization, and also the topological symmetry, in a way that generalizes the Yang-Mills case [1].

The expansion in ghost number of eq. (3.7) and of its Bianchi identity defines the complete topological BRST symmetry. Eq. (3.7) also shows that the topological ghosts and ghosts of ghosts, which appear in its right hand-side, are components of a curvature. This turns out to be the important observation in view of a geometrical interpretation of the gravitino as curvature and not as a gauge field for a gauge symmetry. This interpretation can be generalized to the antighost sector,by introducing a tri-grading, made of the sum of ordinary form degree, the ghost number and the antighost number.

It was found in [6] that the expression of the topological gravity action is:

$$\int_{M_8} \mathcal{L} = \int_{M_8} s\left[\bar{\Psi}^{(-1)ac^-}(b^{(0)cb^-} + \omega^{cb^-}(e) - G^{cb^-}_d e^d)\mathcal{V}_{ab} + \bar{\chi}^{(-1)a}\left(b^{(0)}_a + \partial_a\sigma + \Omega_{abcd}G_{bcd}\right)\right] + \\ + s\left[\partial_{[\mu}A^{(-2)}_{\nu]^-}(\Psi^{(1)a}_{[\mu} e^a_{\nu]} + \Psi^{(1)}_{\mu\nu})\right] s\left[e^\mu_c e^\nu_d\ \bar{\tilde{\Phi}}^{(-2)cd^+}(\Psi^{(1)}_{\mu\nu} - \Psi^{(1)a}_{[\mu} e^a_{\nu]})\right], \tag{3.8}$$

where $\mathcal{V}_{a_1\ldots a_i} \equiv \frac{1}{(8-i)!}\epsilon_{a_1\ldots a_8} e^{a_{(i+1)}}\ldots e^{a_8}$. This topological action gives kinetic terms for the graviton, the gravitino, the 2-form $B_{\mu\nu}$, both two graviphotons $A^{\pm 2}_\mu$ and the dilaton and dilatino of the of $N = 1$ $D = 8$ supergravity, around an octonionic gravitational instanton and a given configuration of the dilaton. It was emphasized in [6] that, by suitable deformations of the topological gauge functions, it is possible to reproduce the exact supergravity action.

To gauge-fix the local supersymmetry, which emerges as an invariance of the topological ghosts in the action (3.8), we added in [6] the following s-exact action:

$$\int_{M_8} s\left[\sqrt{g}(\bar{\Phi}^{(-2)a} D_\mu \Psi^{(1)a}_\mu + \Phi^{(0)ab^-} D_\mu \bar{\Psi}^{(-1)ab^-}_\mu + \bar{\Phi}^{(0)}\partial_\mu \bar{\Psi}^{(-1)}_\mu)\right]. \tag{3.9}$$

This expression is quite instructive to us. It allows us to respectively identify $(\Phi^{(2)a}, \Phi^{(0)ab-}, \Phi^{(0)})$, $(\bar{\Phi}^{(-2)a}, \tilde{\Phi}^{(0)ab-}, \bar{\Phi}^{(0)})$ and $(\bar{\eta}^{(-1)a}, \eta^{(1)ab^-}, \eta^{(1)})$ as twisted versions of the Faddeev-Popov spinor ghost and antighost for local $N = 1$ supersymmetry, and of the fermionic Lagrange multipliers for the gauge-fixing of the gravitino, with a gauge function $(\gamma^\mu\partial_\mu)\gamma^\nu\lambda_\nu$.

The gauge-fixing of both graviphotons $A^{\pm 2}_\mu$ was obtained from:

$$\int_{M_8} s(S^1\partial_\mu A^{-2}_\mu + R^{-3}\partial_\mu A^2_\mu). \tag{3.10}$$

Thus, (d^1, S^{-1}) and (R^{-3}, R^3) can be identified as the Faddeev-Popov ghosts and antighosts for the U(1) invariances of both graviphotons.

The mapping between the fermionic degrees of freedom between the fermionic sector of the TQFT and of $D = 8$, $N = 1$ supergravity uses the covariantly constant spinor ε of the manifold with Spin(7) holonomy. Call respectively $(\lambda, \bar{\lambda})$ and $(\chi, \bar{\chi})$ the chiral and antichiral parts of the gravitino and dilatino. Their relation with the topological ghosts of the TQFT was found to be:[4]

$$
\begin{aligned}
\lambda &= \Psi^a \gamma_a \varepsilon \,, \\
\bar\lambda &= \bar\Psi \varepsilon + \bar\Psi^{ab^-} \gamma_{ab} \varepsilon \,, \\
\chi &= \bar\chi^a \gamma^a \varepsilon \,, \\
\bar\chi &= \chi \varepsilon + \chi^{ab^-} \gamma_{ab} \varepsilon \,.
\end{aligned}
\tag{3.11}
$$

3.2 Completion of the spectrum of $N = 1$ $D = 8$ supergravity by the TQFT of a 3-form

The previous section section was for recalling the link between $N = 1$ $D = 8$ supergravity and a TQFT that involves the fields in eq. (3.1). We now show the novel result that the construction of a TQFT for a 3-form and its coupling to the TQFT that we have just written, determines $N = 2$ $D = 8$ supergravity.

In order to reach the $N = 1$ $D = 8$ supergravity, we have set to zero in a BRST invariant way the fields $A^{(0)}_{\mu\nu^-}$ and $A^{(0)}$, which appear in the TQFT multiplet in eq. (3.4). We used a term $s[\bar\Psi^{(-1)}_{\mu\nu^-} A^{(0)}_{\mu\nu^-} + \bar S^{(-1)} A^{(0)}]$ [6]. Here, we will use our freedom of possibly defining a propagation for these fields, by a change of the topological gauge functions, and reach eventually the $N = 2$ $D = 8$ supergravity.

We thus go back to the situation where the degrees of freedom in $A^{(0)}_{\mu\nu^-}$ and $A^{(0)}$ are arranged as the elements of a vector ghost of ghost $A^{(0)}_\mu$. The latter will be interpreted shortly as the twisted expression of the chiral part of a commuting spinor, which turns out to be the Faddeev-Popov antighost for the second generator of $N = 2$ supersymmetry. This suggests that we need new fields, for representing this symmetry. The natural idea is to introduce a 3-form gauge field $C_{\mu\nu\rho}$, together with its topological multiplet, and to complete the $N = 1$ theory into the $N = 2$ theory, by addition of a TQFT for the 3-form.

We now must find the way to write the BRST multiplet for a 3-form. Following the notation that is analogous to that we have used for the BRST multiplet of the 2-form $B_{\mu\nu}$, the BRST multiplet of the 3-form $C_{\mu\nu\rho}$ is:

$$
\begin{array}{ccccccccccccccccc}
&&&&&&&& C_{\mu\nu\rho} &&&&&&&& \\
&&&&&&& \swarrow &&&&&&&&& \\
&&&&&& \psi^{(1)}_{\mu\nu\rho} &&&& \bar\psi^{(-1)}_{\mu\nu\rho\sigma+} &&&&&& \\
&&&&& \swarrow &&&& \swarrow &&&&&&& \\
&&&& B^{(2)}_{\mu\nu} &&&& B^{(0)}_{\mu\nu^-}, b^{(0)}_{\mu\nu\rho\sigma+} &&&& B^{(-2)}_{\mu\nu} &&&& \\
&&& \swarrow &&&& \swarrow &&&& \swarrow &&&&& \\
&& R^{(3)}_\mu &&&& S^{(1)}_\mu, \psi^{(1)}_{\mu\nu^-} &&&& S^{(-1)}_\mu, \psi^{(-1)}_{\mu\nu} &&&& R^{(-3)}_\mu && \\
& \swarrow &&&& \swarrow &&&& \swarrow &&&& \swarrow &&& \\
M^{(4)} &&&& N^{(2)}, b^{(2)}_\mu &&&& N^{(0)}, b^{(0)}_\mu &&&& N^{(-2)}, b^{(-2)}_\mu &&&& M^{(-4)} \\
&&& \swarrow &&&& \swarrow &&&& \swarrow &&&& \swarrow & \\
&& \eta^{(3)} &&&& \eta^{(1)} &&&& \eta^{(-1)} &&&& \eta^{(-3)} &&
\end{array}
\tag{3.12}
$$

[4]The 8-dimensional gamma matrices γ_a act on spinors of definite chirality. We have defined $\chi_{\mu\nu^-} = \Psi^{(1)}_{\mu\nu^-} - \Psi^{(1)a}_{[\mu} e^a_{\nu]^-}$ and $\chi_{ab^-} = e^\mu_a e^\nu_b \chi_{\mu\nu}$ [6].

The BRST equations for all these fields can be obtained in an analogous way as for those the 2-form, with equations as in eq. (3.7).

In eq. (3.12) the topological antighost of the 3-form has been already identified as a self-dual 4-form $\bar\psi_{\mu\nu\rho\sigma+}$. This can be accepted as an input. Alternatively, we can start from an earlier stage and define the 56 components of the topological antighost of C_3 in a Spin(7)-invariant way as $35 \oplus 21$. Then, we can eliminate the 21-component against the component $B^{(0)}_{\mu\nu+}$ of the ghost of ghost $B^{(0)}_{\mu\nu}$, with a term of the form $s(\bar\psi^{(-1)}_{\mu\nu+} B^{(0)}_{\mu\nu+})$. Eventually, this explains the absence of the forms $\bar\psi^{(-1)}_{\mu\nu+}$, $B^{(0)}_{\mu\nu+}$, $b^{(0)}_{\mu\nu+}$ and $\psi^{(1)}_{\mu\nu+}$ among the fields that we have displayed in the table (3.12). This reduction is natural, since we basically need a self-dual Lagrange multiplier 4-form $b^{(0)}_{\mu\nu\rho\sigma+}$ for determining the square of the 4-form curvature $G_4 = dC_3$ of the 3-form C_3, using a standard self-duality gauge function $G_4 =^* G_4$, and $\int_{M^8} |G_4^+|^2 = \int_{M^8}(|G_4|^2 + G_4 \wedge G_4)$.

As a generalization of what we did for the vector ghost of ghost of the Kalb-Ramond 2-form, we must introduce the fields that are relevant to gain control of the gauge invariance of the 2-form ghost of ghost $B^{(-2)}_{\mu\nu}$. We thus replace the field $\bar\Psi^{(-1)}_{\mu\nu}$ within eq. (3.12) by the following more refined set of fields:

$$
\begin{array}{ccccccc}
\psi^{(-1)}_{\mu\nu} & \hookrightarrow & & & \psi^{(-1)}_{\mu\nu}, \bar R^{(-1)}_{\mu} & & \\
& & & \swarrow & & & \\
& & \Phi^{(0)}_{\mu} & & & & \bar\Phi^{(0)}_{\mu} \\
& & & & & \swarrow & \\
& & & & \eta^{(1)}_{\mu} & &
\end{array}
\tag{3.13}
$$

where

$$
\begin{array}{ccccccc}
\Phi^{(0)}_{\mu} & \hookrightarrow & & & \Phi^{(0)}_{\mu}, m^{(0)} & & \\
& & & \swarrow & & & \\
& & c^{(1)} & & & & \bar c^{(-1)} \\
& & & & & \swarrow & \\
& & & & \bar X^{(0)} & &
\end{array}
\tag{3.14}
$$

$$
\begin{array}{ccccccc}
\bar\Phi^{(0)}_{\mu} & \hookrightarrow & & & \bar\Phi^{(0)}_{\mu}, \bar m^{(0)} & & \\
& & & \swarrow & & & \\
& & \bar c^{(1)} & & & & c^{(-1)} \\
& & & & & \swarrow & \\
& & & & X^{(0)} & &
\end{array}
\tag{3.15}
$$

$$
\begin{array}{ccccccc}
\eta^{(1)}_{\mu} & \hookrightarrow & & & \eta^{(1)}_{\mu}, \bar\eta^{(1)}_{(X)} & & \\
& & & \swarrow & & & \\
& & X^{(2)} & & & & \bar X^{(-2)} \\
& & & & & \swarrow & \\
& & & & \eta^{(-1)}_{(X)} & &
\end{array}
\tag{3.16}
$$

These substitutions amount to the addition of BRST quartets that count altogether for zero degrees of freedom, and can be cast in the general framework of unification between form degrees, ghost number and antighost number. The propagation of these fields will be given shortly, as well as their interpretation.

Obtaining the second gravitino of the theory from the BRST multiplet of the 3-form is less intuitive than the first gravitino. As shown in eq. (3.11), the first gravitino is a rather obvious combination of Spin(7) covariant topological ghosts of the metric and of the Kalb-Ramond 2-form. Our claim is that the chiral and antichiral components of the second gravitino are obtained by untwisting the following sets of forms, which come from the topological multiplet of the 3-form:

$$\left(\bar{\psi}^{(-1)}_{\mu\nu\rho\sigma^+}, \psi^{(-1)}_{\mu\nu}, \eta^{(-1)}_{(X)}\right) \qquad \left(\psi^{(1)}_{\mu\nu\rho}, \eta^{(1)}_{\mu}\right). \tag{3.17}$$

Each one of these combinations has 64 components that are irreducibly decomposed as $35\oplus21\oplus7\oplus1$ under Spin(7) and as $56\oplus8$ under SO(8). By appropriately multiplying these tensors by gamma matrices, and applying the resulting 8×8 matrices on the covariantly constant spinor ϵ of the Spin(7) holonomy manifold, one gets chiral and antichiral spinors:

$$\begin{aligned} \lambda'_\mu &= \left(\bar{\psi}^{(-1)}_{\mu\nu\rho\sigma^+}\gamma^{\nu\rho\sigma} + \psi^{(-1)}_{\mu\nu}\gamma^{\nu} + \eta^{(-1)}_{(X)}\gamma_\mu\right)\epsilon \\ \bar{\lambda}'_\mu &= \left(\psi^{(1)}_{\mu\nu\rho}\gamma^{\nu\rho} + \eta^{(1)}_{\mu}\right)\epsilon \end{aligned} \tag{3.18}$$

These spinors determine a combination of a gauge-fixed gravitino with a Higgsino. We will be more precise on this decomposition shortly.

We now start to define the lagrangian of all these fields, in the tree approximation. Eventually, this will allow for their physical interpretation. We determine the kinetic energy of the second gravitino of the action by the sum of the following BRST-exact terms:

$$\begin{aligned} \int s\left(\bar{\psi}^{(-1)}_{\mu\nu\rho\sigma^+}\left(\frac{1}{2}b^{(0)}_{\mu\nu\rho\sigma^+} + \partial_{[\mu}C_{\nu\rho\sigma]}\right)\right) &= \int \frac{1}{2}|\partial_{[\mu}C_{\nu\rho\sigma]}|^2 - \bar{\psi}^{(-1)}_{\mu\nu\rho\sigma^+}\partial_{[\mu}\psi^{(1)}_{\nu\rho\sigma]} \\ \int s\left(\psi^{(1)}_{\mu\nu\rho}\partial_{[\mu}B^{(-2)}_{\nu\rho]}\right) &= \int \partial_{[\mu}B^{(2)}_{\nu\rho]}\partial_{[\mu}B^{(-2)}_{\nu\rho]} - \bar{\psi}^{(1)}_{\mu\nu\rho}\partial_{[\mu}\psi^{(-1)}_{\nu\rho]} \\ \int s\left(\psi^{(-1)}_{\mu\nu}\partial_{[\mu}\bar{\Phi}^{(0)}_{\nu]}\right) &= \int \partial_{[\mu}\Phi^{(0)}_{\nu]}\partial_{[\mu}\bar{\Phi}^{(0)}_{\nu]} - \psi^{(-1)}_{\mu\nu}\partial_{[\mu}\eta^{(1)}_{\nu]} \\ \int s\left(\bar{X}^{(-2)}\partial_\mu\eta^{(1)}_\mu\right) &= \int \bar{X}^{(-2)}\partial^\mu\partial_\mu X^{(2)} - \eta^{(-1)}_{X^2}\partial_\mu\eta^{(1)}_\mu\,. \end{aligned} \tag{3.19}$$

The sum of the fermionic terms is eqs. (3.19) gives the Rarita-Schwinger action for the second gravitino, expressed in a twisted form, and with an algebraic gauge condition of the type $\gamma^\mu\lambda_\mu = 0$. The compensating Faddeev-Popov ghost dependent action will be determined shortly.

The bosonic terms that are present in the right-hand sides of eqs. (3.19) also indicate that we are on the right track for determining $N = 2$ $D = 8$ supergravity. They provide gauge invariant kinetic energy for two 2-forms, $B^{(2)}_{\mu\nu}$ and $B^{(-2)}_{\mu\nu}$, two graviphotons, $\Phi^{(0)}_\mu$ and $\bar{\Phi}^{(0)}_\mu$, and two scalars, $\bar{X}^{(-2)}$ and $X^{(2)}$.

The dependence on the 3-form C_3 is equivariant with respect to the tensor gauge invariance $C_3 \sim C_2 + d\Lambda_2$, so it is only through the curvature dC_3. However, the topological field theory BRST construction provides all necessary fields for gauge-fixing the local symmetries of the topological ghosts ghosts of ghosts of C_3.

We must gauge-fix $B^{(2)}_{\mu\nu}$ and $B^{(-2)}_{\mu\nu}$, whose classical action has already been obtained in the second line of eqs. (3.19). This is done from the following term:

$$\begin{aligned}\int s\left(S^{(1)}_\mu(\partial_\nu B^{(-2)}_{\mu\nu} + \partial_\mu N^{(-2)}) + R^{(-3)}_\mu(\partial_\nu B^{(2)}_{\mu\nu} + \partial_\mu N^{(2)}) + S^{(1)}_\mu b^{(-2)}_\mu\right) = \\ = \int\Bigg(\partial_\nu B^{(-2)}_{\mu\nu}\partial_\rho B^{(2)}_{\mu\rho} + \bar N^{(-2)}\partial^\mu\partial_\mu N^{(2)} + \partial_{[\mu}S^{(1)}_{\nu]}\partial_{[\mu}R^{(-1)}_{\nu]} + S^{(1)}_\mu\left(\partial_\nu\psi^{(-1)}_{\mu\nu} + \partial_\mu\eta^{(-1)}\right) + \\ + \partial_{[\mu}R^{(3)}_{\nu]}\partial_{[\mu}R^{(-3)}_{\nu]} + R^{(-3)}_\mu\partial_\mu\eta^{(3)}\Bigg). \qquad (3.20)\end{aligned}$$

This expression shows us that the propagators of vectors ghosts must be gauge-fixed, so we add the further term:

$$\begin{aligned}\int s\left(\bar M^{(-4)}\partial_\mu R^{(3)}_\mu + \bar m^{(0)}\partial_\mu R^{(-1)}_\mu\right) = \int\Bigg(\bar M^{(-4)}\partial^\mu\partial_\mu M^{(4)} + \bar m^{(0)}\partial^\mu\partial_\mu m^{(0)} + \bar m^{(0)}\partial_\mu\Phi^{(0)}_\mu + \\ + \bar\eta^{(-3)}\partial_\mu R^{(3)}_\mu + \bar c^{(1)}\partial_\mu R^{(-1)}_\mu\Bigg) \qquad (3.21)\end{aligned}$$

We now understand that $(R^{(3)}_\mu, R^{(-3)}_\mu)$ and $(R^{(-1)}_\mu, S^{(1)}_\mu)$ are respectively standard Faddeev-Popov vector ghosts and antighosts for the covariant gauge-fixing of both 2-form gauge fields $B^{(\pm 2)}_{\mu\nu}$.[5] The fields $M^{(\pm 4)}$, $N^{(\pm 2)}$ and $(m^{(0)}, m^{(0)})$ are the standard ghosts of ghosts of these vector ghosts. Thus, the sum of both actions in eqs. (3.20) and (3.21) is nothing but the covariant gauge-fixing lagrangian for both 2-forms $B^{(\pm 2)}_{\mu\nu}$ in a generalized Feynmann gauge, including all relevant ghosts for the vector gauge symmetry of 2-forms. This result shows the efficiency of the TQFT construction when it comes to encoding all gauge symmetries of the fields.

A remaining task is that of gauge-fixing the abelian symmetries of $\bar\Phi^{(0)}_\mu$ and $\Phi^{(0)}_\mu$. This is done by writing:

$$\begin{aligned}\int s\left(\bar c^{(-1)}\partial_\mu\Phi^{(0)}_\mu + c^{(-1)}\partial_\mu\bar\Phi^{(0)}_\mu + \bar c^{(-1)}X^{(0)}\right) = \\ = \int\left(\bar c^{(-1)}\partial^\mu\partial_\mu c^{(1)} + c^{(-1)}\partial^\mu\partial_\mu\bar c^{(1)} + \partial_\mu\bar\Phi^{(0)}_\mu\partial_\nu\Phi^{(0)}_\nu\right) \qquad (3.22)\end{aligned}$$

In the last equation, $X^{(0)}$ and $\bar X^{(0)}$ have been eliminated by gaussian integration. This gives the gauge-fixing action for $\bar\Phi^{(0)}_\mu$ and $\Phi^{(0)}_\mu$ in the Feynman gauge. $(\bar c^{(1)}, c^{(-1)})$ and $(c^{(1)}, \bar c^{(-1)})$ in eq. (3.14) are thus identified respectively as the Fadeev-Popov ghosts and antighosts for the abelian invariance of $\bar\Phi^{(0)}_\mu$ and $\Phi^{(0)}_\mu$.

The propagation of the fields $(B^{(0)}_{\mu\nu^-}, N^{(0)}, b^{(0)}_\mu, S^{(-1)}_\mu, \psi^{(1)}_{\mu\nu^-}, \eta^{(1)})$ of the 3-form TQFT multiplet remains to be defined, which we now do.

[5] We must redefine $\eta^{(1)}_\mu \to \eta^{(1)}_\mu + S^{(1)}_\mu$ to absorb the term $S^{(1)}_\mu\partial_\nu\psi^{(-1)}_{\mu\nu}$ in eq. (3.20). Analogously, we redefine $\bar X^{(0)} \to \bar X^{(0)} + \bar m^{(0)}$ to absorb the term $\bar m^{(0)}\partial_\mu\Phi^{(0)}_\mu$.

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The fields $(B^{(0)}_{\mu\nu^-}, N^{(0)})$ determine up to twist the chiral part of a commuting Majorana spinor, which we will identify as the Faddeev-Popov ghost of the second generator of local supersymmetry in 8 dimensions. We consider the action:

$$\begin{aligned}&\int s\left(S^{(-1)}_\mu\left(\partial_\nu B^{(0)}_{\mu\nu^-}+\partial_\mu N^{(0)}\right)\right)=\\&\qquad=\int S^{(-1)}_\mu\left(\partial_\nu\psi^{(1)}_{\mu\nu^-}+\partial_\mu\eta^{(1)}\right)+b^{(0)}_\mu\left(\partial_\nu B^{(0)}_{\mu\nu^-}+\partial_\mu N^{(0)}\right). \qquad (3.23)\end{aligned}$$

The last term is identified as the ghost part of a Faddeev-Popov action, for a gauge condition that sets eight components of the second gravitino equal to zero. These eight components build the chiral part of the spin one-half component of the second gravitino. We are actually gradually unveiling the second local supersymmetry. The first term in eq. (3.23) is a Dirac lagrangian for the anticommuting Majorana spinor that one obtains by untwisting $(S^{(-1)}_\mu, \psi^{(1)}_{\mu\nu^-}, \eta^{(1)})$. This field is one Higgsino of the supergravity.

The gauge-fixing term of the antichiral part of the second gravitino involves, as already suggested, the fields $(A^{(0)}_{\mu\nu^-}, A^{(0)}) \sim A^{(0)}_\mu$ of the spectrum of the 2-form B_2 in eq. (3.2). These fields were set equal to zero in [6], so as to obtain $N=1$ $D=8$ supergravity. We now choose another gauge-fixing for defining the TQFT, and define a propagation for these commuting ghosts, by means of the action:

$$\begin{aligned}&\int s\left(A^{(0)}_\mu\left(\partial_\nu\Psi^{(-1)}_{\mu\nu^-}+\partial_\mu\bar{S}^{(-1)}\right)\right)=\\&\qquad=\int \Psi^{(1)}_\mu\left(\partial_\nu\psi^{(-1)}_{\mu\nu^-}+\partial_\mu S^{(-1)}\right)+A^{(0)}_\mu\left(\partial_\nu b^{(0)}_{\mu\nu^-}+\partial_\mu b^{(0)}\right). \qquad (3.24)\end{aligned}$$

The last term is indeed the desired Faddeev-Popov term for the antichiral part of the second supersymmetry. The first term in eq. (3.24) is another twisted Higgsino term. The sum of both second terms in the actions (3.23) and (3.24) determine a complete Faddeev-Popov ghost action for a gauge condition $\Gamma^\mu\lambda_\mu=0$, where λ stand for the chiral and antichiral ungauged-fixed components of the second gravitino. We conclude that, in a twisted form, the Faddeev-Popov ghosts and antighosts for the second component of local $N=2$ supersymmetry are:

$$\left(b^{(0)}_\mu, B^{(0)}_{\mu\nu^-}, N^{(0)}\right) \qquad \left(A^{(0)}_\mu, b^{(0)}_{\mu\nu^-}, b^{(0)}\right). \qquad (3.25)$$

Of course, one may feel puzzled by the fact that the TQFT chooses different gauge conditions for the gravitinos of both sectors of $N=2$ local supersymmetry. This is actually explainable, since both gravitinos originate as topological ghosts of different objects, a 2-form and a 3-form. This interesting inelegance might disappear, if we are able to covariantly formulate our equations in higher dimensions, perhaps in nine dimensions. In the untwisted model, it is however possible to readjust the gauge, in such a way that the symmetry of both supersymmetry sectors is recovered.

As for the number of degrees of freedom in the TQFT, the gauge-fixing that is revealed by the actions in eqs. (3.23) and (3.24) suggests that only 56 components of each one of the multiplets in eq. (3.17) are really part of the second gravitino.

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We are now in the position of more precisely identifying the fields in eq. (3.17). They must be understood as made of the twisted gauge-fixed second gravitino and of one independent Majorana spinor, $(\eta^{(1)}_\mu, \psi^{(-1)}_{\mu\nu^-}, \eta^{(-1)}_{(X)})$. The latter will be a third Higgsino of the supergravity, according to:

$$\begin{aligned} \left(\bar\psi^{(-1)}_{\mu\nu\rho\sigma+}, \psi^{(-1)}_{\mu\nu}, \eta^{(-1)}_{(X)}\right) &\to \left(\bar\psi^{(-1)}_{\mu\nu\rho\sigma+}, \psi^{(-1)}_{\mu\nu^+}\right) \oplus \left(\psi^{(-1)}_{\mu\nu^-}, \eta^{(-1)}_{(X)}\right) \\ \left(\psi^{(1)}_{\mu\nu\rho}, \eta^{(1)}_\mu\right) &\to \psi^{(1)}_{\mu\nu\rho} \oplus \eta^{(1)}_\mu \,. \end{aligned} \tag{3.26}$$

We should now rest and contemplate the field content of the theory that we have constructed step by step.

The idea was to define a propagation for all fields of the BRST multiplet of a 3-form, and to complete the TQFT of [6]. The TQFT sector stemming from a 2-tensor (the metric and the Kalb-Ramond 2-form) yields a propagating theory for the vielbein, one scalar (the dilaton), the 2-form, two vectors (two abelian graviphotons), one Majorana gravitino and one Majorana spinor, according to:

$$e^a_\mu, B_{\mu\nu} \to e^a_\mu, B_{\mu\nu}, \sigma, A^{(2)}_\mu, A^{(-2)}_\mu, \left(\Psi^{(1)a}_\mu, \bar\Psi^{(-1)ab^-}_\mu, \bar\Psi^{(-1)}_\mu\right), \left(\bar\chi^{(-1)}_\mu, \Psi^{(1)}_{\mu\nu^-}, \chi^{(1)}\right). \tag{3.27}$$

The TQFT sector stemming from the 3-form yields a second Majorana gravitino in a twisted form, two abelian 2-forms, two vectors, two scalars and three Majorana spinors, according to:

$$\begin{aligned} C_{\mu\nu\rho} \to\; & C_{\mu\nu\rho}, B^{(\pm 2)}_{\mu\nu}, \bar\Phi^{(0)}_\mu, \Phi^{(0)}_\mu, X^{(2)}, \bar X^{(-2)}, \left(\bar\psi^{(-1)}_{\mu\nu\rho\sigma+}, \psi^{(-1)}_{\mu\nu^+}, \psi^{(1)}_{\mu\nu\rho}\right), \\ & \left(\eta^{(1)}_\mu, \psi^{(-1)}_{\mu\nu^-}, \eta^{(-1)}_{(X)}\right), \left(S^{(-1)}_\mu, \psi^{(1)}_{\mu\nu^-}, \eta^{(1)}\right), \left(\Psi^{(1)}_\mu, \psi^{(-1)}_{\mu\nu^-}, S^{(-1)}\right) \end{aligned} \tag{3.28}$$

The second gravitino is obtained in a gauge of the type $\gamma_\mu \Lambda_\mu = 0$.

The rest of the fields, which appear in the topological multiplets of the 2-form and 3-form multiplets, play the role of ordinary Faddeev-Popov ghosts for gauge-fixing all gauge symmetries of these fields that can be identified as classical fields of $N = 2$ $D = 8$ supergravity. We left aside the gauge-fixing of the 2-form $B_{\mu\nu}$ and 3-form $C_{\mu\nu\rho}$, for the symmetry $C_3 \sim C_3 + dc_2$ and $B_2 \sim B_2 + dc_1$, which is standard in the equivariant construction, as it is already well understood in the context of the topological Yang-Mills theory.

Putting everything together, we have therefore constructed a theory whose physical fields are, up to twist, a metric, a 3-form, three 2-forms, four 1-forms, three scalars, two gravitinos and four Majorana spinors.

This is not yet the complete $N = 2$ $D = 8$ supergravity. However, we left aside the possibility of completing the TQFT multiplet of the 3-form by topological sets that involve two 1-forms, $\bar\Phi^{(-2)}_\mu$ and $\Phi^{(2)}_\mu$, for gaining control to the possible gauge symmetry of $B^{(0)}_{\mu\nu}$. The fields $\bar\Phi^{(2)}_\mu$ and $\bar\Phi^{(-2)}_\mu$ play a role for $B^{(0)}_{\mu\nu}$ that is analogous to that played by $\bar\Phi^{(0)}_\mu$ and $\Phi^{(0)}_\mu$ for $B^{(-2)}_{\mu\nu}$.[6]

[6] A dissymmetry occurs however in the TQFT, because the anti-self part $B^{(0)}_{\mu\nu+}$ of $B^{(0)}_{\mu\nu}$ is set equal to zero, while all components of $B^{(-2)}_{\mu\nu}$ propagate.

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We introduce $\bar{\Phi}^{(-2)}_\mu$ and $\Phi^{(2)}_\mu$ as parts of topological multiplets, whose components count altogether for zero degree of freedom. Such multiplets are generically of the form $(A_\mu, \Psi_\mu, \eta, \kappa_{\mu\nu^-}, \phi, \bar{\phi})$. They must be completed with Faddeev-Popov ghost and antighost and a Lagrange multiplier, $(g, \bar{g}, h)$. The TQFT action that encodes the abelian invariance is $\int s(\kappa_{\mu\nu^-}(H_{\mu\nu^-} + \partial_{[\mu} A_{\nu]^-}) + \eta \partial_\mu \Psi_\mu + \bar{g}(\partial_\mu A_\mu + h))$, which a twisted form of the eight dimensional supersymmetric Yang-Mills theory [1].

We can therefore incorporate in the topological multiplet of the 3-form, which is given in eqs .(3.12)–(3.16), both following topological submultiplets;

$$
\begin{array}{ccccccc}
 & & & \Phi^{(-2)}_\mu & & & \\
 & & \swarrow & & & & \\
 & \psi^{(-1)}_\mu, g^{(-1)} & & & \chi^{(1)}_{\mu\nu^-}, \bar{g}^{(1)} & & \\
 & \swarrow & & & \swarrow & & \\
\phi^{(0)} & & & b^{(2)}_{\mu\nu^-}, h^{(2)} & & & \bar{\phi}^{(0)} \\
 & & & & & \swarrow & \\
 & & & & \bar{\eta}^{(1)} & & \\
 & & & & & & \\
 & & & \Phi^{(2)}_\mu & & & \\
 & & \swarrow & & & & \\
 & \psi^{(3)}_\mu, g^{(3)} & & & \chi^{(-3)}_{\mu\nu^-}, \bar{g}^{(-3)} & & \\
 & \swarrow & & & \swarrow & & \\
\phi^{(4)} & & & b^{(-2)}_{\mu\nu^-}, h^{(-2)} & & & \bar{\phi}^{(-4)}, \\
 & & & & & \swarrow & \\
 & & & & \bar{\eta}^{(-3)} & &
\end{array}
\tag{3.29}
$$

The TQFT action of these fields is made of two twisted super Yang-Mills actions. We add each of them to the TQFT actions that we have already constructed.

We now face with a theory whose physical field content is made of the fields in eq. (3.27)–(3.28), plus two graviphotons, two Higgsinos and four scalars, which stem from the fields that we just introduced in eqs. (3.29). Thus, up to twist, our gravitational TQFT predicts the following set of fields, which are defined modulo ordinary gauge invariances:

$$g_{\mu\nu}, C_{\mu\nu\rho}, 3B_{\mu\nu}, 6A_\mu, 7S, 2\lambda, 6\chi \,. \tag{3.30}$$

The notation is that S, λ and χ respectively denote scalar fields, Majorana gravitinos and Higgsinos. This set of fields fields is nothing, but the spectrum [10] of $N = 2$ $D = 8$ supergravity. The lagrangians of the TQFT and of supergravity coincide in the quadratic approximation for the fermions.

All other fields that have occurred in our construction can be understood as conventional ghosts and Lagrange multipliers for fixing the gauge symmetries of the fields in eq. (3.30). The way all fields transform under the TQFT symmetry has been determined from the tri-complex structure that one associates to the gauge symmetries of gravity coupled to a 2-form and a 3-form. The expression of the topological BRST transformations can be reinterpreted to reproduce the symmetries of $N = 2$ $D = 8$ supergravity, including local supersymmetry.

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The fields that are displayed in eq. (3.30) can also be understood as the spectrum of $N = 1$, $D = 11$ supergravity. Indeed, the straightforward dimensional reduction of $N = 1$, $D = 11$ supergravity gives:

$$(g_{\mu\nu}, C_{\mu\nu\rho}, \lambda)_{D=11} \sim (g_{\mu\nu}, C_{\mu\nu\rho}, 3B_{\mu\nu}, 6A_\mu, 7S, 2\lambda, 6\kappa)_{D=8} \,. \tag{3.31}$$

To conclude this section, let us insist on the following points. As for the identification of the TQFT action and the supergravity action, our demonstration holds at the quadratic level for the fermions. Suitable modifications of the gauge functions by higher order terms should reveal the exact laws of supergravity. Moreover, the word identification means that the gravitational TQFT reproduces the supergravity around a Spin(7) $\subset$ SO(8) invariant background. The SU(2) internal symmetry of the $N = 2$ $D = 8$ supergravity multiplet is not explicit in our construction. This is a direct consequence of the fact that, in the 8-dimensional TQFT, both gravitinos have different origins, as topological ghosts of a 2-form of a 3-form.

4. A digression about the TQFT for the 3-form

It is actually an interesting question to investigate whether a TQFT for the 3-form alone has some interest, in particular for the study of 8-dimensional manifolds with Spin(7) holonomies and of 7-dimensional manifolds with G_2 holonomies. This was suggested as early as in [1]. We would like to briefly sketch new ideas relative to this question and indicate a possible hint for getting an interacting theory for the 3-form, when topological gravity is decoupled.

We have seen in the previous section that the TQFT multiplet for a 3-form is basically made of the following fields:

$$C_3, 2B_2, 2A_1, 2S, \lambda_\mu^\alpha, 2\chi^\alpha \,. \tag{4.1}$$

These fields are those that propagate in the TQFT, provided one does a suitable gauge-fixing. Spinorial notations have already been used for the Fermi fields that are topological ghosts.

We understood in the previous section that the gravitino λ_μ^α (a 1-form which a spinorial index) is made of topological ghosts, including the self-dual antighost $\chi_{\mu\nu\rho\sigma^+}$ of C_3 The multiplet (4.1) does not involve a metric. If we follow the conventional pattern for building a TQFT, we introduce an external metric $g_{\mu\nu}$, and do a topological BRST invariant gauge-fixing. In this way, we can get a free TQFT action for the fields in eq. (4.1). The action contains the term:

$$\begin{aligned} \int d^8x s \left(\chi^{\mu\nu\rho\sigma^+} \left(\partial_{[\sigma} B_{\mu\nu\rho]} + H_{\mu\nu\rho\sigma^+} \right) \right) &= \int d^8x \left(|dB_3 +^* dB_3|^2 + \cdots \right) \\ &= \int d^8x \sqrt{g} \left(g^{\mu\mu'} g^{\nu\nu'} g^{\rho\rho'} g^{\sigma\sigma'} \partial_{[\sigma} B_{\mu\nu\rho]} \partial_{[\sigma'} B_{\mu'\nu'\rho']} + \cdots \right) . \end{aligned} \tag{4.2}$$

The dots are easy to compute. We can dimensionally reduce the fields and get a TQFT in seven dimensions, for a 3-form and a 2-form. The self-duality condition in eight dimensions is replaced by the following one in seven dimensions, which also stands for 35 conditions:

$$dC_3 +^* dB_2 = 0\,. \tag{4.3}$$

Imposing these conditions in a BRST-invariant way is allowed by the fact that C_3 and B_2 carry 20 and 15 degrees of freedom modulo gauge transformations in seven dimensions. Eq. (4.3) is a generalization of the well-known Bogomolny equation.

If we can define a metric $g_{ij}(B_3)$ in seven dimensions, which depends on the 3-form, and use it for the * Hodge operation, we can trigger interactions between the forms, by defining the following action:

$$\begin{aligned}
&\int d^7xs \left(\chi^{ijk}((dB_2 +^* dC_3)_{ijk} + H_{ijk})\right) \\
&\quad = \int d^7x(|dB_2 +^* dC_3|^2 + \cdots) \qquad\qquad (4.4)\\
&\quad = \int d^7x\sqrt{g}\left(g^{ii'}g^{jj'}g^{kk'}\partial_{[i}B_{jk]}\partial_{[i'}B_{j'k']} + g^{ii'}g^{jj'}g^{kk'}g^{ll'}\,\partial_{[i}C_{jkl]}\partial_{[i'}C_{j'k'l']} + \cdots\right).
\end{aligned}$$

The dots involve the metric dependence of the 3-form. This action is analogous in spirit to that of supersymmetric mechanics in curved space. The subtlety is to find the relevant configuration space of the 3-form for defining the path integral.

A further reduction in six dimensions determines theories with self-dual 2-forms. Such TQFTs have been directly studied in six dimensions, and interesting correspondences with supersymmetry have already been found [13].

There is another topological field theory, which we can obtain by directly starting from the topological action $\int_{M_7} C_3 \wedge dC_3$. For certain manifolds, one can give a special role to the direction x^7 and write:

$$\int_{M_7} C_3 \wedge dC_3 = \int_{M_7} d^7x \left(C_{7ab}\epsilon^{abcdef}\partial_{[c}C_{def]} + \epsilon^{abcdef}C_{abc}\partial_7 C_{def}\right). \tag{4.5}$$

Here, a, b, c, d, e, f stand for 6-dimensional indices.

Formal manipulations, which generalize those for the analysis of the Chern-Simons theory in three dimensions [12], suggest that the 7-dimensional theory with the above action might also describe a theory in six dimensions, depending of a 2-form, with an action of the type:

$$\int_{M_6} d^6x\sqrt{g}g^{aa'}g^{bb'}g^{cc'}\partial_{[a}B_{bc]}\partial_{[a'}B_{b'c']} + \text{WZZ action}\,. \tag{4.6}$$

A heuristic argument can be done by formally integrating out C_{7ab}, solving the constraint $\partial_{[a}B_{bc]} = 0$, and picking a slice at $x^7 = cte$. Eventually, the 6d-metric depends on the 2-form, $g^{aa'} = g^{aa'}(dB_2)$, and we have a non trivial model. The observables in the seven dimensional theory theory are obtained from the flux of the 3-form on a 3-cycle:

$$\exp \int\int\int_{\Gamma_3} C_{ijk}dx^i \wedge dx^j \wedge dx^k \tag{4.7}$$

and

$$\exp \int\int_{\Gamma_2} B_{ij}dx^i \wedge dx^j\,. \tag{4.8}$$

We leave the investigation of these theories for future work.

5. Higher dimensions and $N = 1$ $D = 11$ supergravity

We have shown in section 3 that the 8-dimensional gravitational TQFT reproduces, up to twist, the $N = 2$ $D = 8$ supergravity theory in the quadratic approximation in the gravitino fields, around a Spin(7) invariant vacuum. However, we left open the question of precisely determining the mechanism according to which Chern-Simons interactions are built in the TQFT.

We wish to look in more details how we can lift in higher dimensions the 8-dimensional TQFT that we have built in section 3.

Let us recall that $N = 1$ $D = 11$ supergravity can be reduced to type IIA 10-dimensional supergravity, which, in turns, can be reduced to a 9-dimensional model and, eventually, to $N = 2$ $D = 8$ supergravity theory.

This 9-dimensional model can also be deduced from type IIB 10-dimensional supergravity. The 9-dimensional theory is interesting due the remarkable properties of Spin(9) [17].

To relate the 8-dimensions TQFT with another one in 9 dimensions, we tentatively refer to the mechanism explained in [16]. There, it is shown on general grounds that, if one has a TQFT in d dimensions, with an action of the type $\mathcal{I}_d = \int_{M_d}(d(...) + \{Q, \ldots\})$, where Q^2 vanishes modulo some gauge transformations, then, it exists another operator Q_{d+1} in $d + 1$ dimensions, such that

$$Q^2_{d+1} = \partial_{x^{d+1}} , \tag{5.1}$$

modulo gauge transformations. Q_{d+1} acts on d+1 dimensional fields which are in one to one correspondence with those of the d-dimensional theory. Q_{d+1} is identical to Q when fields are taken at equal values of x^{d+1}. The existence of the Q-exact action $\mathcal{I}_d$ implies that of an action $\mathcal{I}_{d+1}$ in $d + 1$ dimensions, which is of the type [16]:

$$\mathcal{I}_{d+1} = \int_{M_{d+1}} (\Delta_{d+1} + \{Q_{d+1}, \ldots\}) \tag{5.2}$$

Δ_{d+1} contains Chern-Simons terms and is Q_{d+1}-invariant but not Q_{d+1}-exact. It descends from the cocycles that one can construct for the cohomology of the Q symmetry [16]. It contains terms that are only $SO(d) \subset SO(d + 1)$ invariant. These general properties have been analyzed in great details in [16], and geometrical arguments have been given for explaining such a delicate mechanism that relates theories in d and d+1 dimensions.

The second term in the r.h.s. of eq. (5.2) gives back the action $\mathcal{I}_d$ we started from, by standard dimensional reduction, up to Chern-Simons terms. However, as emphasized in in [16], one may obtain deformations of the original d-dimensional theory. For instance, compactification on a circle provides a radius R that may be used as a parameter for the deformation, when one descends from $d + 1$ to d dimensions.

In our case, we start from the the 8 dimensional Q-exact action that we have introduced in section 3. Using the method of [16], we can construct 9-dimensional fields from the 8-dimensional ones and introduce the following Q_9-invariant term in 9 dimen-

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sions:

$$\int \Delta_9 = \int \Big(A \wedge dC_3 \wedge dC_3 + B_2^{(-2)} \wedge dB_2 \wedge dC_3 + dx^9 \times \tag{5.3}$$
$$\times \left(\varphi_0^{(2)} dC_3 \wedge dC_3 + \varphi_1^{(0)} \wedge dB_2 \wedge dC_3 + \psi_3^{(1)} \wedge \psi_3^{(1)} \wedge dA + dB_2^{(-2)} \wedge \Psi_2^{(1)} \wedge \Psi_3^{(1)} \right) \Big) .$$

We can add to Δ_9 a Q_9-exact action, which is inspired from the one we have constructed in section 3. Eventually, it is likely that we will determine in this way the 9-dimensional supergravity action, including Chern-Simons terms.

The first terms in (5.3) are the dimensional reduction, modulo d-exact terms, of the 11-dimensional Chern-Simons term

$$\Delta_{11} = C_3 \wedge dC_3 \wedge dC_3 + dx^{11} \left(\varphi_2^{(2)} \wedge dC_3 \wedge dC_3 + \psi_3^{(1)} \wedge \psi_3^{(1)} \wedge dC_3 \right) . \tag{5.4}$$

We thus postulate the existence of a TQFT in 11 dimensions of a TQFT with an action of the following type:

$$\int \left(C_3 \wedge dC_3 \wedge dC_3 + dx^{11} \left(\varphi_2^{(2)} dC_3 \wedge dC_3 + \psi_3^{(1)} \wedge \psi_3^{(1)} \wedge dC_3 \right) + \{Q_{11}, Z_{11}\} \right) . \tag{5.5}$$

The first terms are Q_{11}-closed but not Q_{11}-exact. The last term, which is Q_{11}-exact, determines propagators and regularizes the theory. We conjecture that the existence of the gauge fermion Z_{11} on 11-dimensional manifolds with Spin(7) holonomy follows from that of the TQFT in 8 dimensions. The action (5.5) should be closely related to that of $N = 1$, $D = 11$ supergravity on special manifolds. Various types of compactifications from 11 to 8 and 7 dimensions may produce interesting deformations.

Our suggestion is that the TQFT point of view is likely to to single out the Chern-Simon part of the supergravity action in 11 dimensions as a Q-invariant but not Q-exact terms, while the rest of the action can be cast in a Q-exact form. The latter part, which includes the Einstein-Hilbert and Rarita-Schwinger actions, is important to regularize the theory and the path integral; however, in the topological phase, only the the Chern-Simons part is truly relevant.

6. Conclusion and discussion

This paper shows that, in eight dimensions, the TQFT for a metric, two 1-forms, a 2-form and a 3-form reproduces in a twisted form (at least in the quadratic approximation) the $N = 2$ $D = 8$ supergravity in a Spin(7)-invariant gravitational background, which is the dimensional reduction of $N = 1$ $D = 11$ supergravity. We found the relevant self-duality equations for defining the TQFT.

This result is quite interesting. In particular, it gives an alternative definition for local supersymmetry. It circumvates the challenge of closing the algebra of infinitesimal transformations of supergravities by mean of auxiliary fields and of getting a group structure for the transformations of supergravity. We now understand the fact that the latter "close modulo equations of motions" as a consequence of the determination of a TQFT by enforcing self-duality equations in a BRST invariant way.

Many dimensional reductions of the TQFT that we have exhibited can be thought of. We foresee theories in seven and six dimensions, where $G_2 \subset \mathrm{Spin}(7) \subset \mathrm{SO}(8)$ plays an important role [18], as well as potentially interesting models in 3, 2, 1 and 0 dimensions. In the latter case, there could be a generalization of the DVV matrix model [14] by gravitational terms, stemming from the topological 8-dimensional action. They might be relevant to the recent works in [15].

The 8-dimensional gravitational theory can be also coupled to a non abelian Yang-Mills theory, which allows us to consider other types of models by dimensional reduction, and perhaps, to find a description of the octonionic superstring [20].

A more speculative idea is that, since $N = 1$ $D = 11$ supergravity seems of a topological origin, it might encode by dimensional reduction the supersymmetric expression of stochastically quantized four dimensional Yang-Mills gauge theories, which have also the structure of a TQFT, and efficiently regularize many of the problems that one encounters in the quantization of 4-dimensional Yang-Mills theories [19].

We added the following remarks.

The construction of a TQFT for a 3-form in eight dimensions can be done independently of the context of topological gravity. It still needs Spin(7) holonomy manifolds, but the metric is no longer an independent field. One gets models of the topological type that depend on a gravitino and seem to contain topological observables. Such models are related to the"Chern-Simons action" $\int C_3 \wedge dC_3$ in seven dimensions.

We also suggested that, on special manifolds, the supergravity action in eleven dimensions can be separated into a Q-closed but not Q-exact term, which is the Chern-Simons part, and a Q-exact term, which defines the propagation of the fields.

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